Time-Optimal Trajectory Generation for Dynamic Vehicles: A Bilevel Optimization Approach

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INTRODUCTION

- Time-optimal trajectory generation for dynamic vehicles is hard because of nonlinear objective and complex constraints.
- We reframe the NLP as a bilevel optimization.
- The lower-level optimization is a nonlinear convex problem, the upper-level optimization is a small scale nonlinear problem.
- Gradients are computed through first-order sensitivity analysis.
- Our method is any-time feasible.
- Numerical experiments show our method performs more robustly than general NLP solvers (SNOPT, IPOPT).

Methodology

- Time-optimal Trajectory Generation

$$\begin{aligned} &\text{minimize } T \\ &\text{subject to } q(t) \in X_{\text{free}}, \forall t \in [0,T] \\ &\qquad q(0) = q_s \\ &\qquad q(T) \in X_{\text{goal}} \\ &\qquad h(q,\dot q,\ddot q,u) = 0 \quad \text{Dynamics} \\ &\qquad f(q,\dot q,\ddot q,u) \le 0 \quad \text{Bounded velocity/acceleration} \end{aligned}$$

- Bilevel[2] Formulation

$$\begin{aligned} &\text{minimize } T \\ &\text{subject to } c(s) \in X_{\text{free}}, \forall s \in [0,1] \\ &\qquad c(0) = q_s \\ &\qquad c(1) \in X_{\text{goal}} \\ &\qquad T \in \text{TOPP}(c, h(\cdot), f(\cdot)) \quad \text{Time-optimal Path Parameterization (TOPP)} \end{aligned}$$

- Bilevel Optimization is solved using Quasi-Newton method.
- Gradient is computed by sensitivity analysis of parametric NLPs[1]: $\nabla_c J^\star = \nabla_c J + \lambda^T \nabla_c f(\cdot) + \nu^T \nabla_c h(\cdot)$
- TOPP: find a time parameterization s along a given geometric path c to achieve minimum traversal time:

$$T = \int_0^1 1dt = \int_{s(0)}^{s(T)} \frac{1}{\dot s} ds = \int_0^1 \frac{1}{\dot s} ds$$

$$\dot q(t) = p'(s)\dot s(t), \qquad \ddot q(t) = p'(s)\ddot s(t) + p''(s)\dot s^2(t)$$

- Constraints on dynamics, velocity ($\dot q(t)$) and acceleration ($\ddot q(t)$) can be imposed in terms of $a = \ddot s$ and $b = \dot s^2$.

Efficient TOPP solver

- A primal-dual interior point method w/ a customized KKT solver
- We use a nonlinear objective: SOCP formulation[3] is inefficient as the size of the problem might be doubled by introducing slacks[4].
- Eliminating variables: to reduce the number of variables, we replace a_i with a linear combination of b_i:

$$a_i = (b_{i+1} - b_i)/2\Delta s_i$$

- Customized KKT solver that exploits the sparsity pattern of the reduced KKT system:

$$\begin{bmatrix} S & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = rhs,$$

where $S = H + \tilde G^T W^{-1} W^{-T} \tilde G$, $\tilde G$ includes the gradients of both linear and nonlinear inequality constraints, W is a diagonal scaling matrix, A is the linear equality matrix.

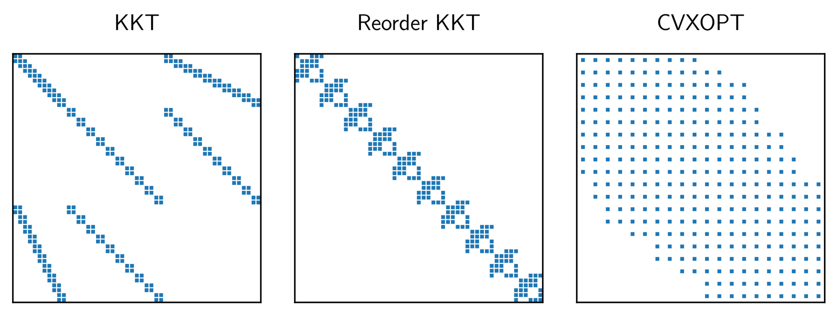

Figure 1. Sparsity pattern of KKT systems. Left to right: the original reduced KKT system, manually reordered reduced KKT matrix with a bandwidth of 6, the L and L^T matrix after LDL factorization used in CVXOPT.

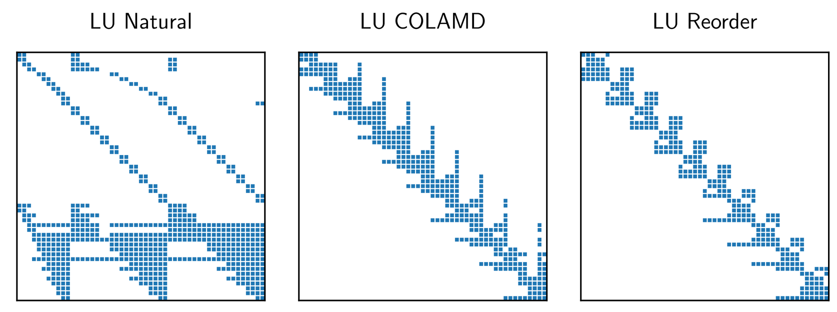

Figure 2. Sparsity after LU factorization. Natural (without reduce fill-in permutation) gives 668 nonzeros,, COLMAD gives 457 nonzeros, our reordering gives 383 nonzeros.

Solver	N = 25	N = 50	N = 100	N = 200
CVXOPT	0.119	0.749	1.30	48.940
Natural + LU	0.011	0.028	0.093	0.414
COLMAD + LU	0.009	0.018	0.035	0.072
Reorder + LU	0.010	0.017	0.032	0.069

Table 1. Comparison of running time for solving the reduced KKT system once (in seconds) on different solvers. Experiment uses 9 convex polygons, each one is discretized into N segments

Experiments

- Friction circle model[4]:

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} \ddot x \\ \ddot y \end{bmatrix}$$

$$\sqrt{u_x^2 + u_y^2} \le \mu g, u_x \le \mu_s \mu g$$

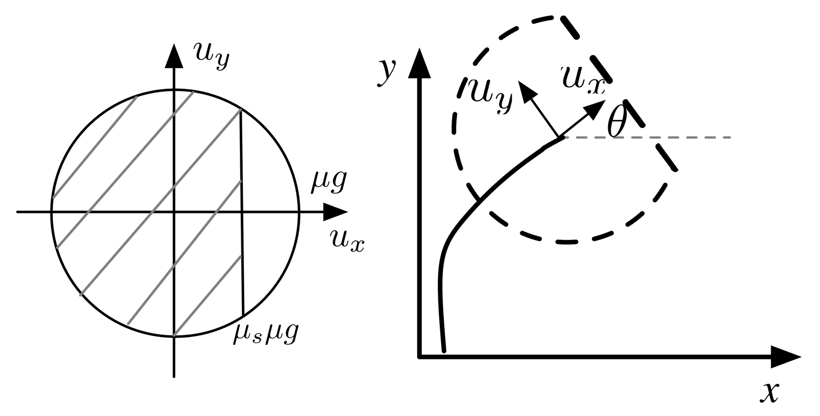

Figure 3. Friction circle model

- Outdoor racing track: Circuit Ricardo Tormo

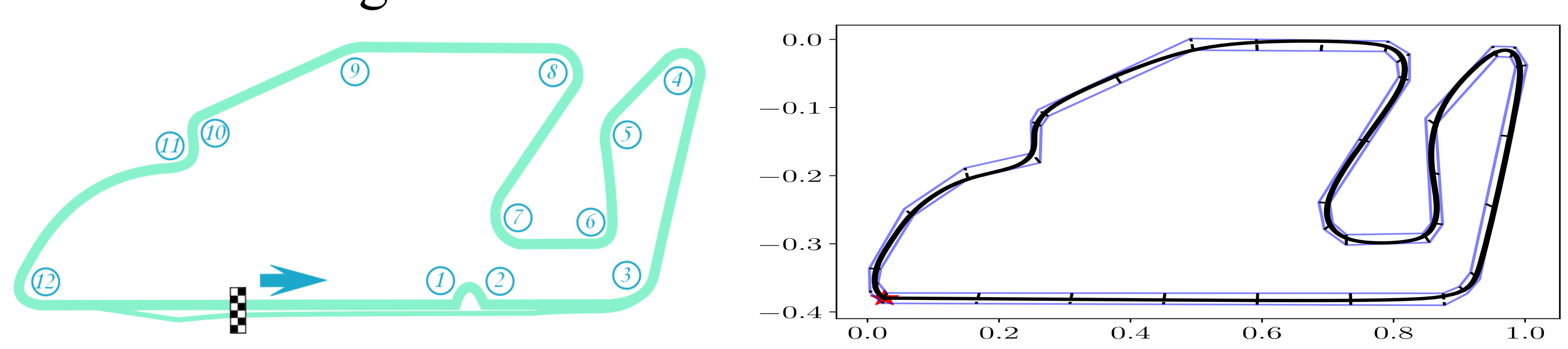

Figure 4. Time-optimal path for Circuit Ricardo Tormo

- Indoor racing track: Tamiya Asia Cup Finals 2011

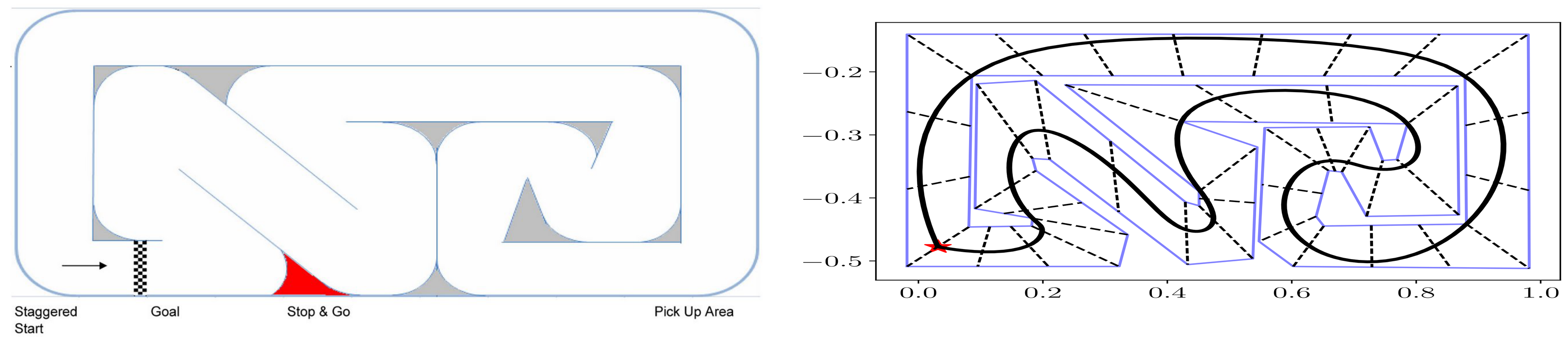

Figure 5. Time-optimal path for Tamiya Asia Cup Finals 2011

- Convergence and constraint violation w.r.t. computation time

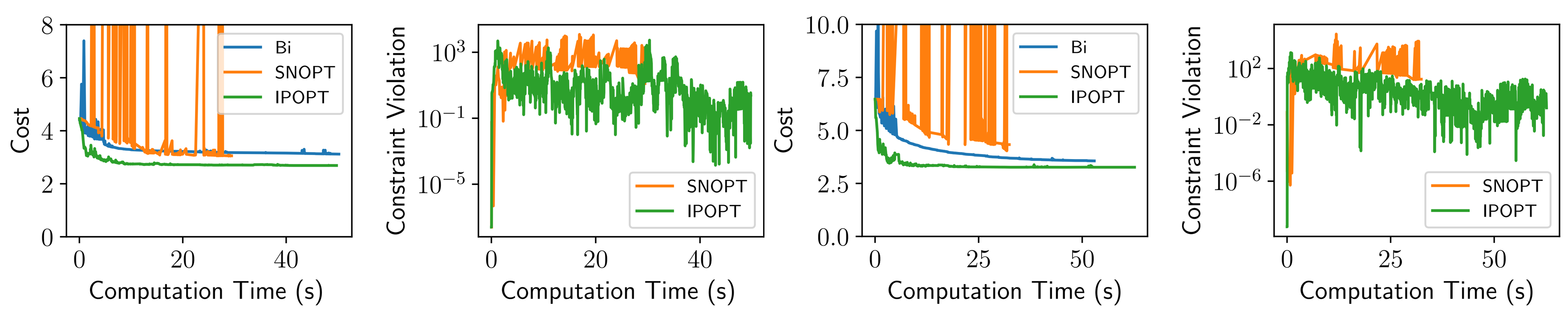

Figure 6. Left: outdoor track, right: indoor track

CONCLUSION

- Our solver is any-time and highly robust than general nonlinear solvers like SNOPT and IPOPT.
- An efficient TOPP solver based on a customized nonlinear convex solver is presented.

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