Fast UAV Trajectory Generation using Bilevel Optimization

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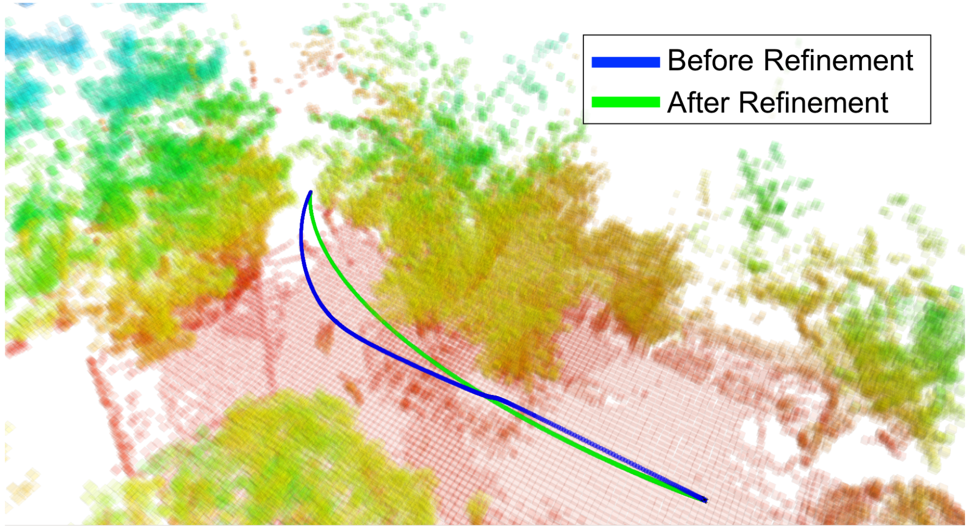
DUKE ROBOTICS

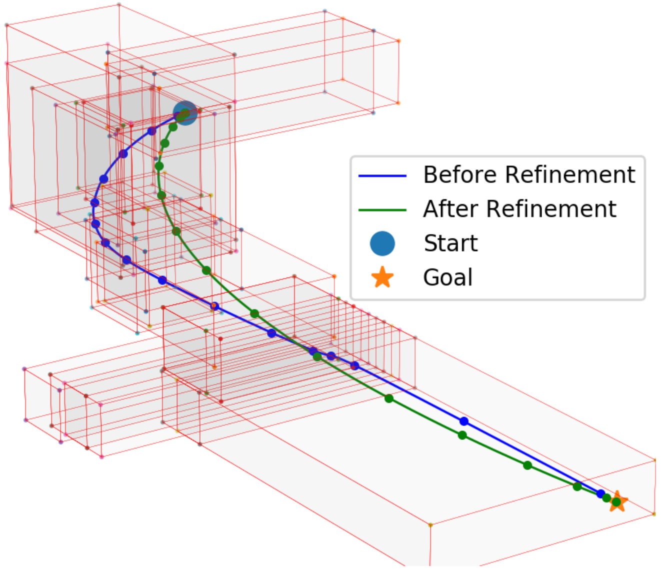
IML Intelligent Motion Lab

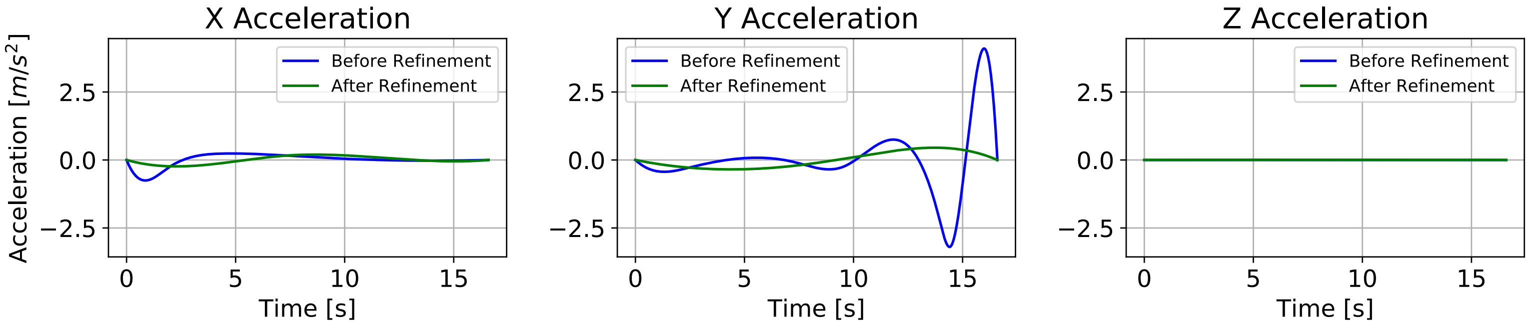
Motivation

- Time allocation for spline trajectories is important, but hard

	Jerk	Length
Before[1]	45.06	20.7m
After	0.15	19.9m







[1] F. Gao et al., "Online safe trajectory generation for quadrotors using fast marching method and bernstein basis polynomial," *ICRA*, 2018

Why hard?

- Time enters optimization nonlinearly
- Time is refined by gradient descent, but gradient is hard to compute.

Formulation

For a flight corridor with n segments, use a piecewise Bézier curve of order d:

$c \in R^{3n(d+1)}$: control points of the curve

$y \in R^{n}_{++}$: time allocation

minimize $J = \underbrace{c^T P(y) c}_{\text{Jerk}} + w \underbrace{\mathbf{1}^T y}_{\text{Traversal time}}$

Quadratic in c
Nonlinear in y

subject to $G(y)c \leq h$

Trajectory stays in flight corridor
Velocity/acceleration stay in the bound

$L(y)c = m$

C^2 continuity at knot points
Trajectory starts/ends at initial/final state

$Ay \leq b, Cy = d$

Fixed traversal time (optional),
Time is positive

Bilevel Formulation

minimize $J = c^T P(y) c + w\, 1^T y$

subject to $G(y) c \leq h$
$L(y) c = m$
$Ay \leq b, Cy = d$

→

minimize $J = c^T P(y) c + w\, 1^T y$

subject to $\mathrm{c} \in \operatorname{argmin} \{J:\ G(y)c \leq h, L(y)c = m\}$
$Ay \leq b, Cy = d$

We use constrained gradient descent:

$$y = y - \alpha \mathrm{proj}_{\mathrm{A,C}}(\nabla_y J^\star(y))$$

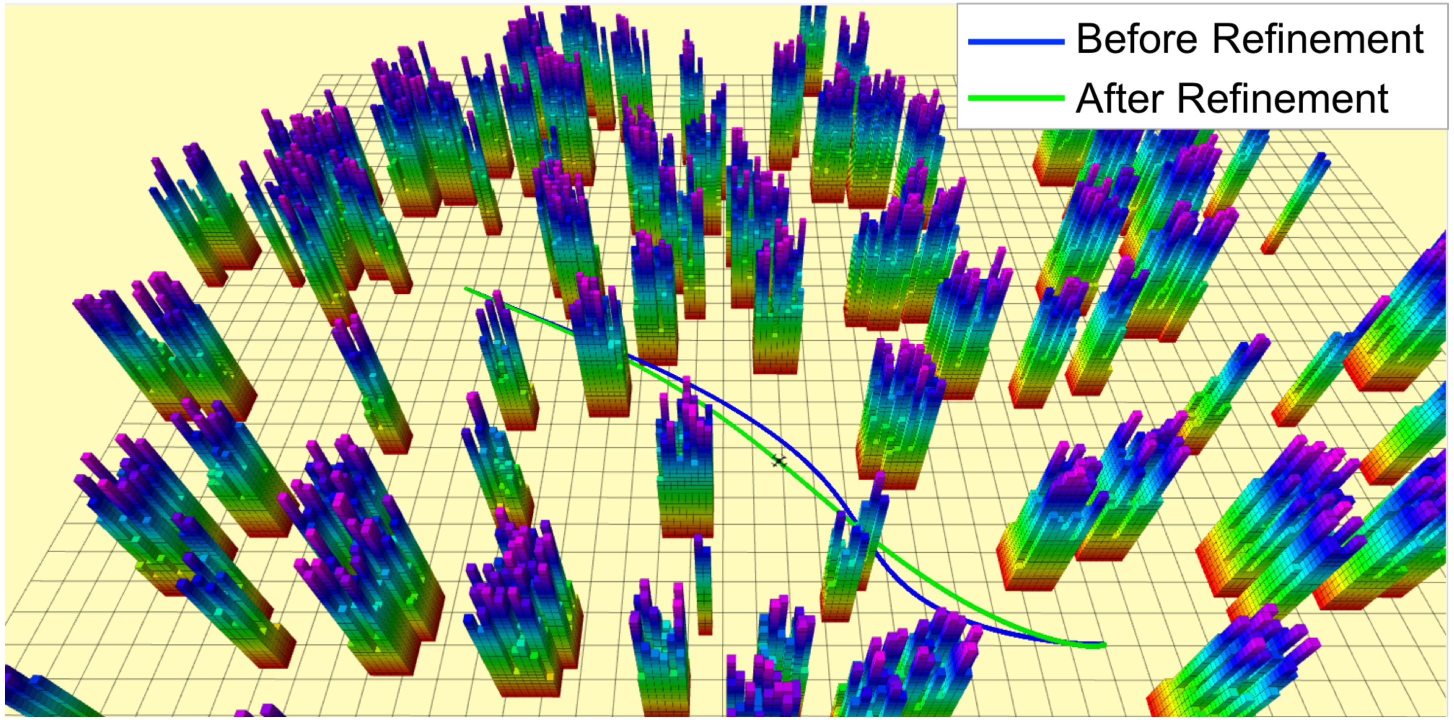
Gradient computation (from sensitivity analysis of parametric NLPs):

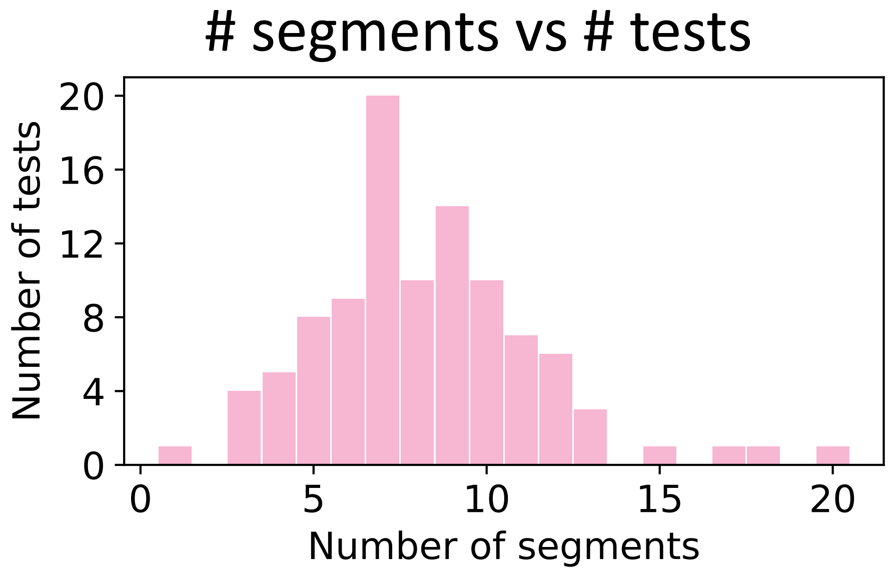
$$\nabla_y J^\star(y) = \nabla_y J + \lambda^T \nabla_y (\, G(y)c - h) + \nu^T \nabla_y (L(y)c\ - m)$$

λ, ν: Lagrange multipliers, which can be obtained “for free” by solving the QP

Numerical experiments: real-time performance

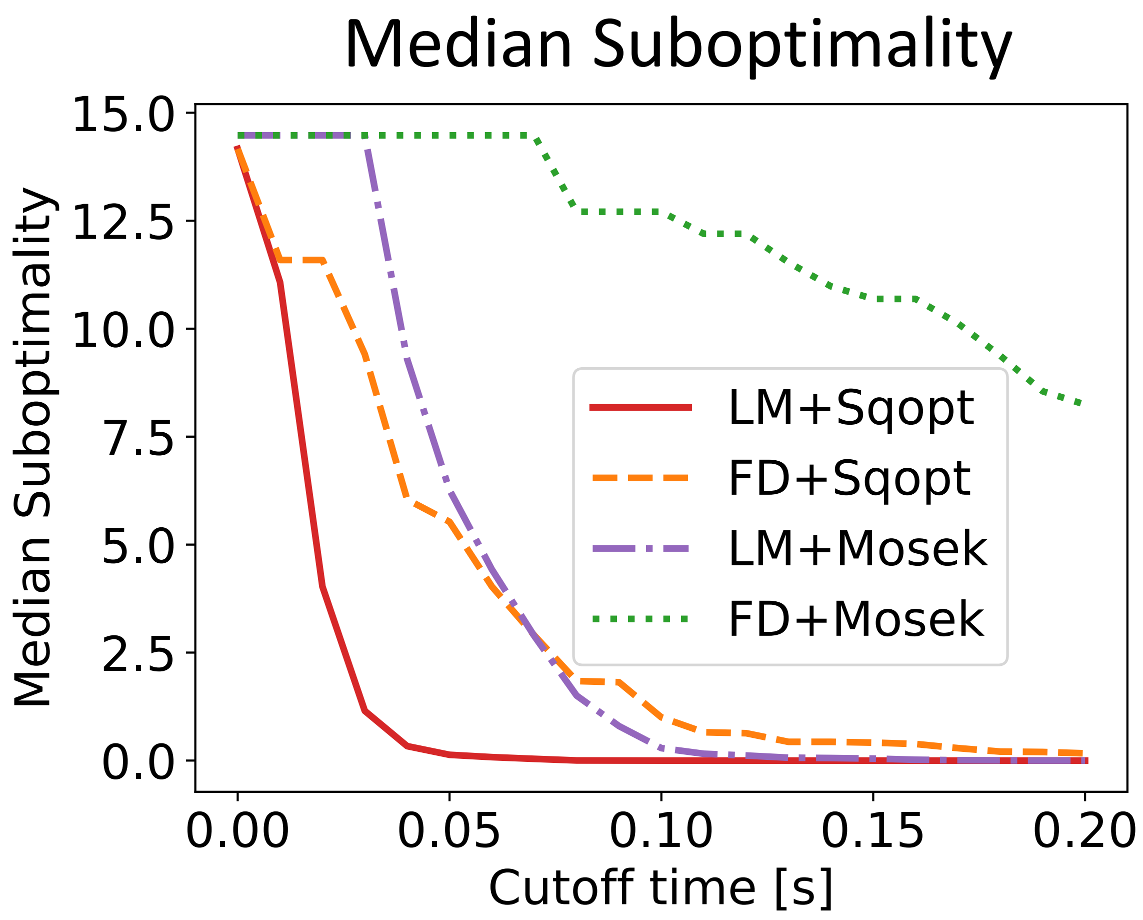
- 100 tests: Random environment + random start/goal, fixed T, $w = 0$.
- We solve c, y to optimal.

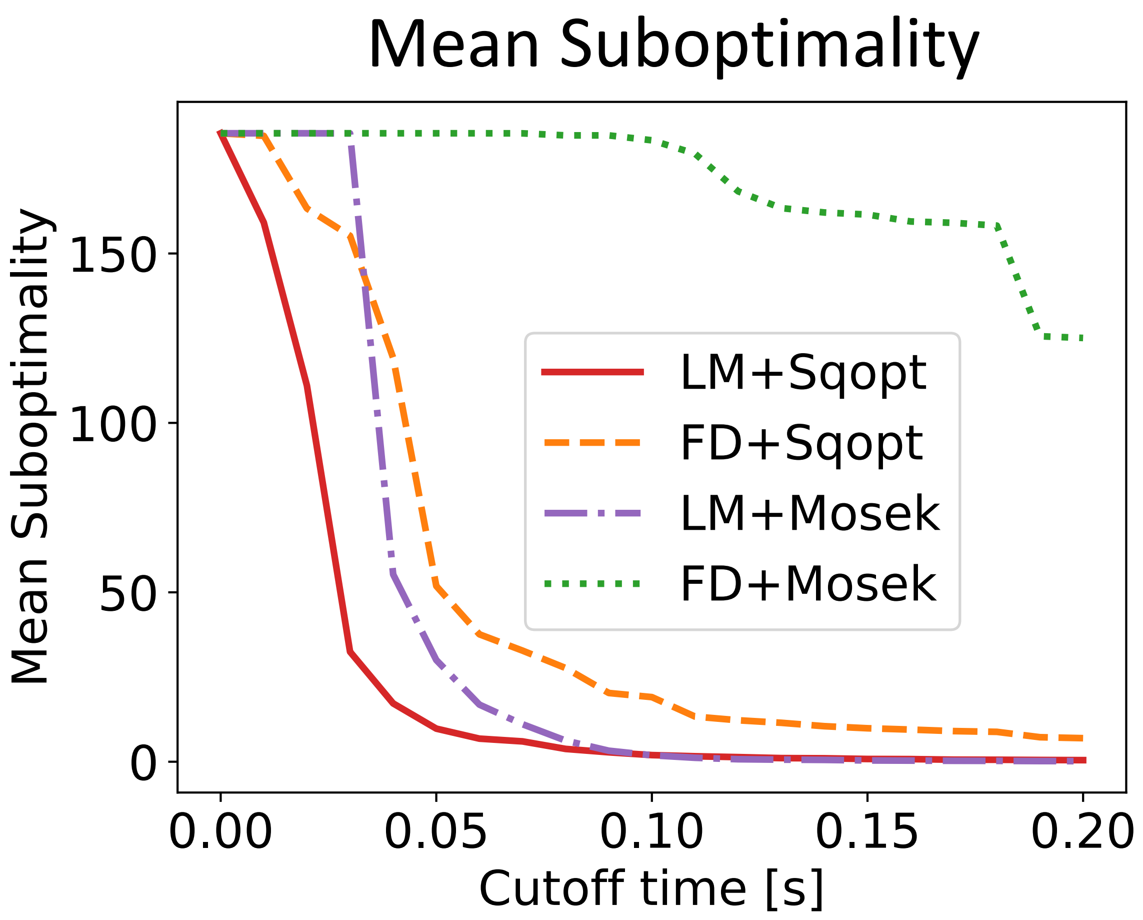




Numerical experiments: real-time performance

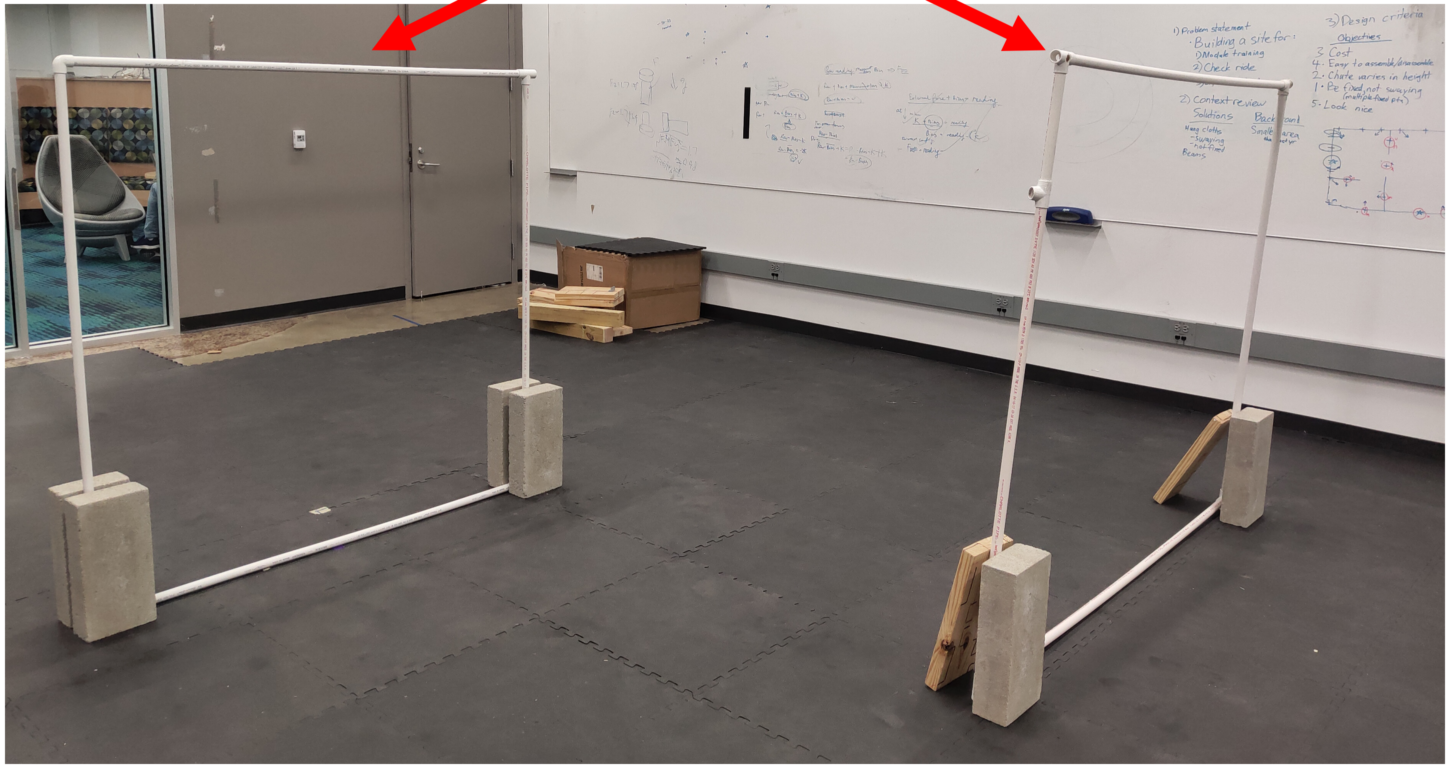
- Our method (LM) vs finite difference (FD)
- 2 QP solvers are used: Sqopt (active-set), Mosek (interior-point)



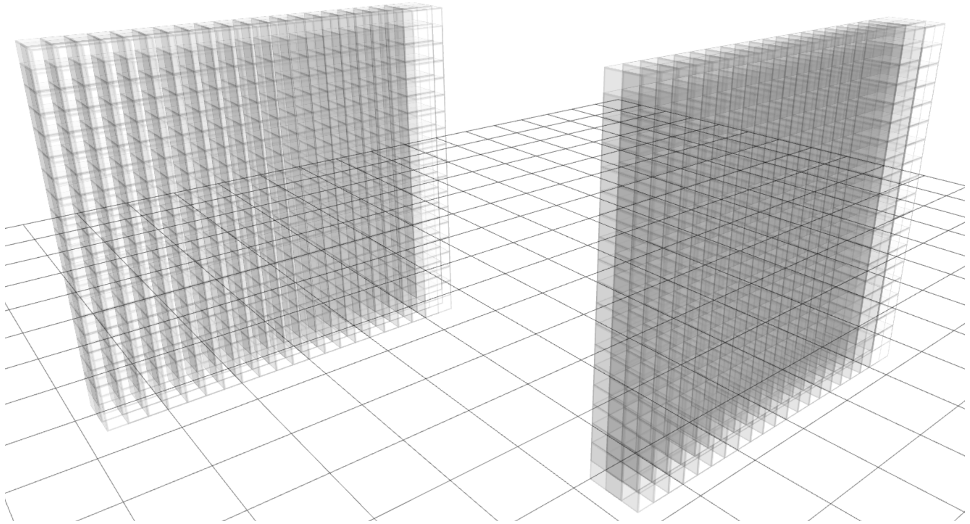


Physical Experiment Setup

Frames treated as walls



Physical layout

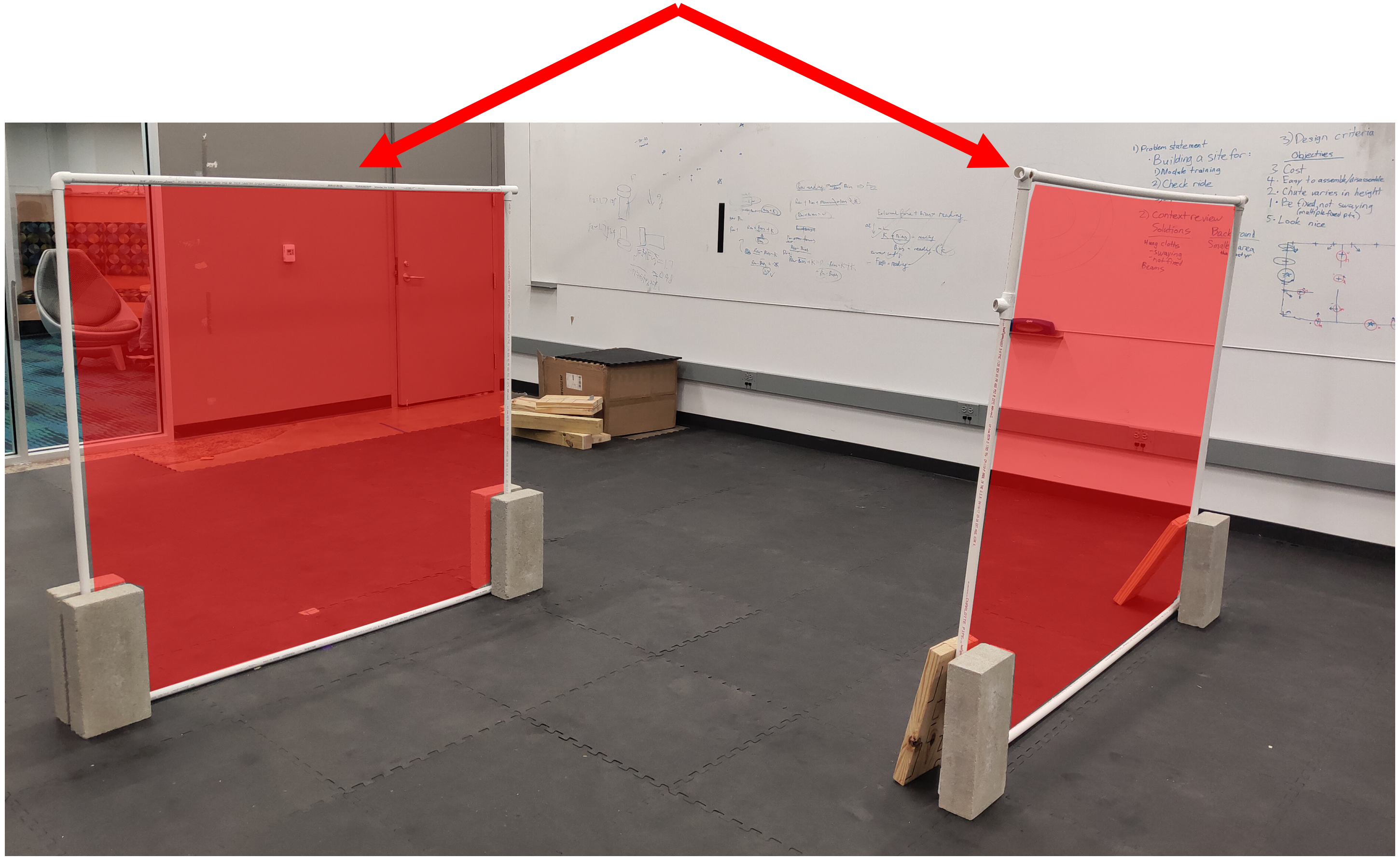


Rviz visualization

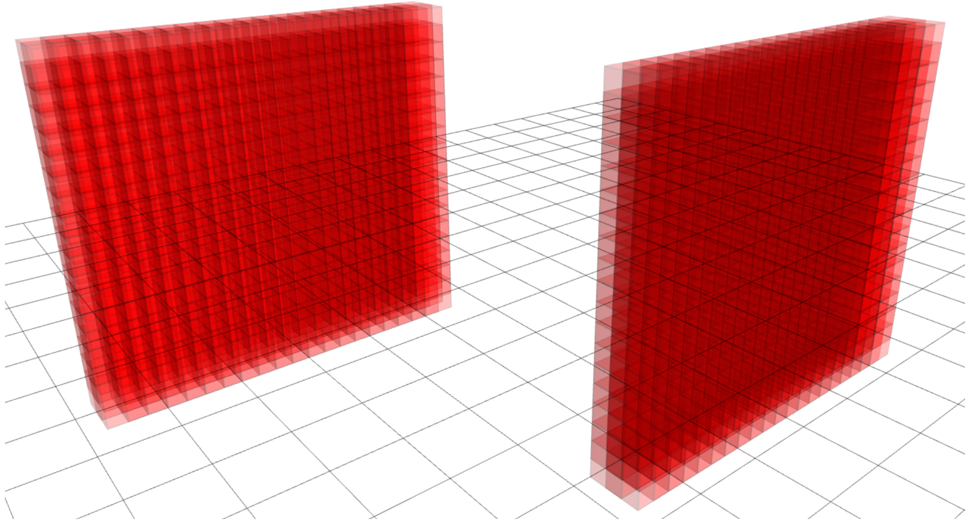
We use the Crazyflie 2.1

Physical Experiment Setup

Frames treated as walls



Physical layout



Rviz visualization

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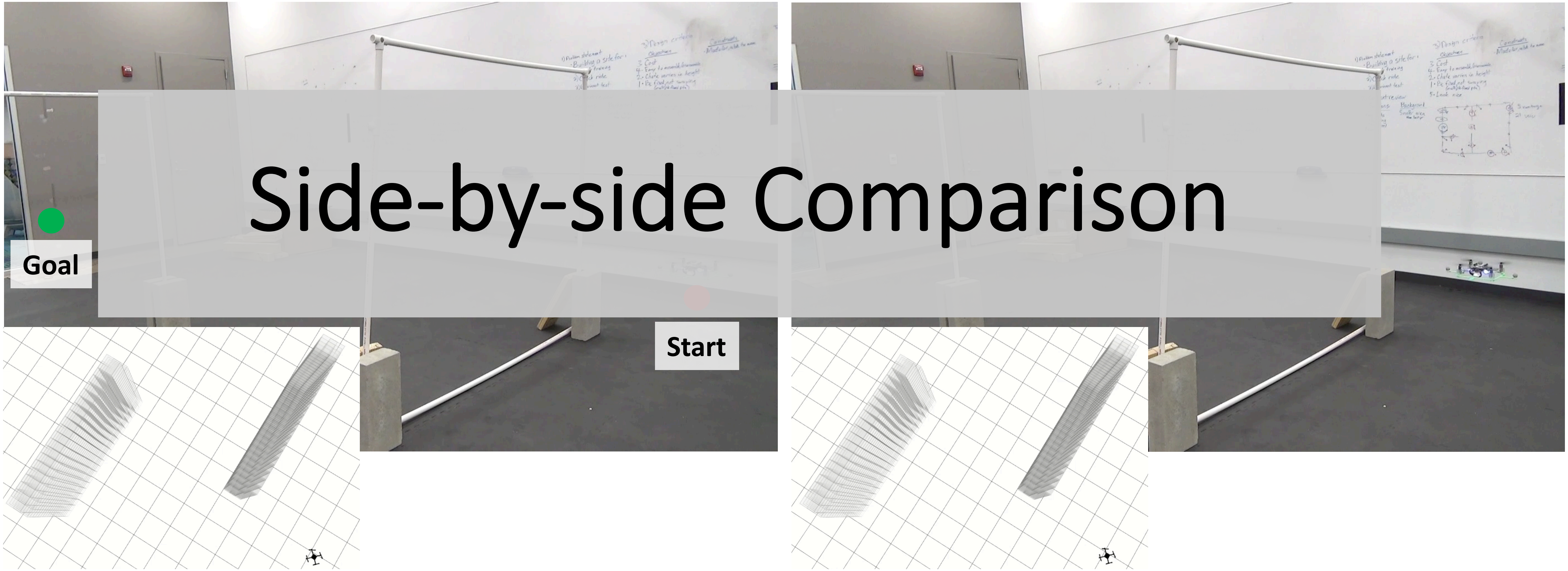
Experiment 1

Our method plans a faster trajectory than state-of-the-art [1] with the same jerk

Gao et al. [1], $T = 5.32$s, Jerk=39

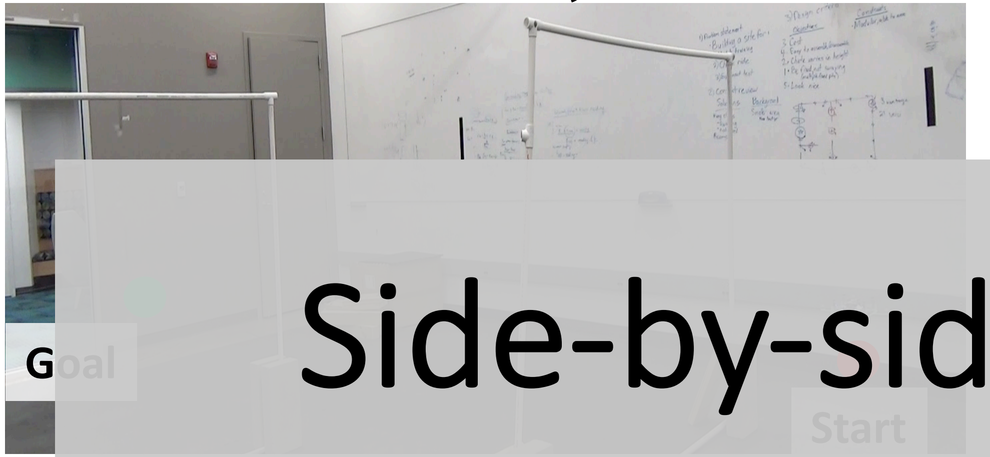
Ours, $T = 4.36$s, Jerk=39



Experiment 2

Our method can control aggressiveness using time penalty w (Plan time ~10 ms)

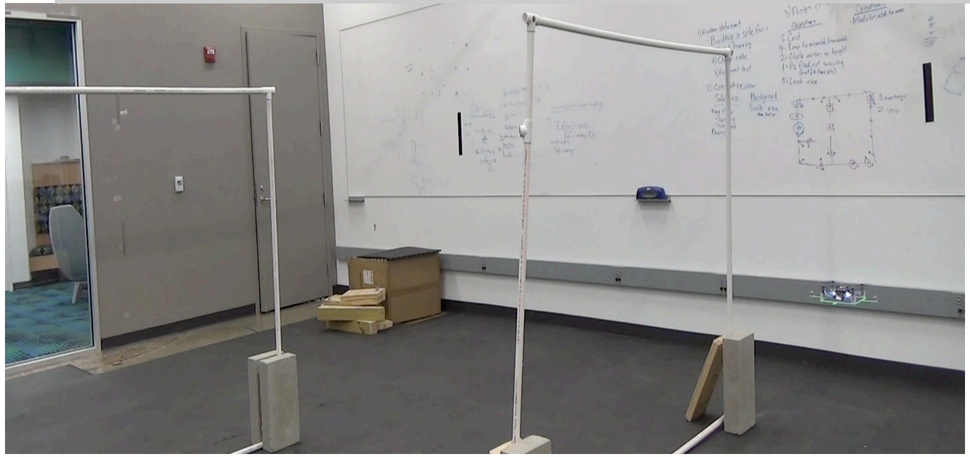
$w = 10, T = 5.60\text{s}, \text{Jerk} = 11.2$



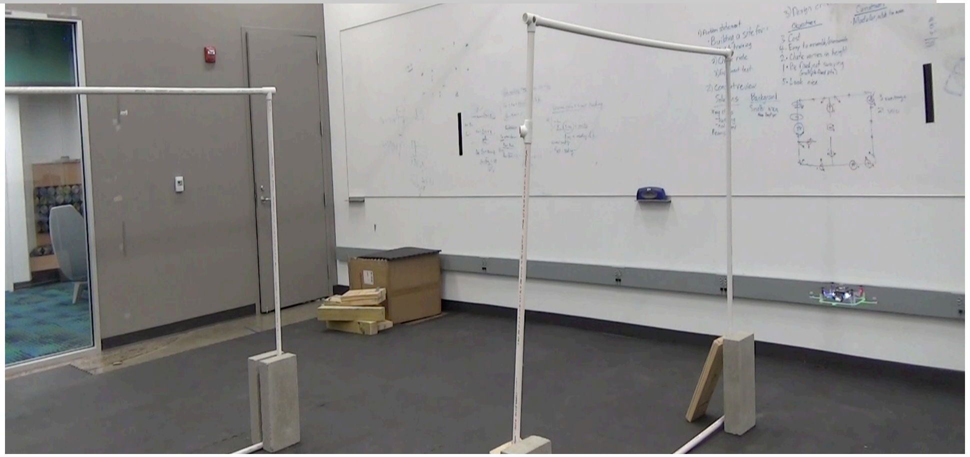
$w = 20, T = 4.96\text{s}, \text{Jerk} = 19.9$

Side-by-side Comparison

$w = 40, T = 4.42\text{s}, \text{Jerk} = 36.1$



$w = 80, T = 4.01\text{s}, \text{Jerk} = 64.7$



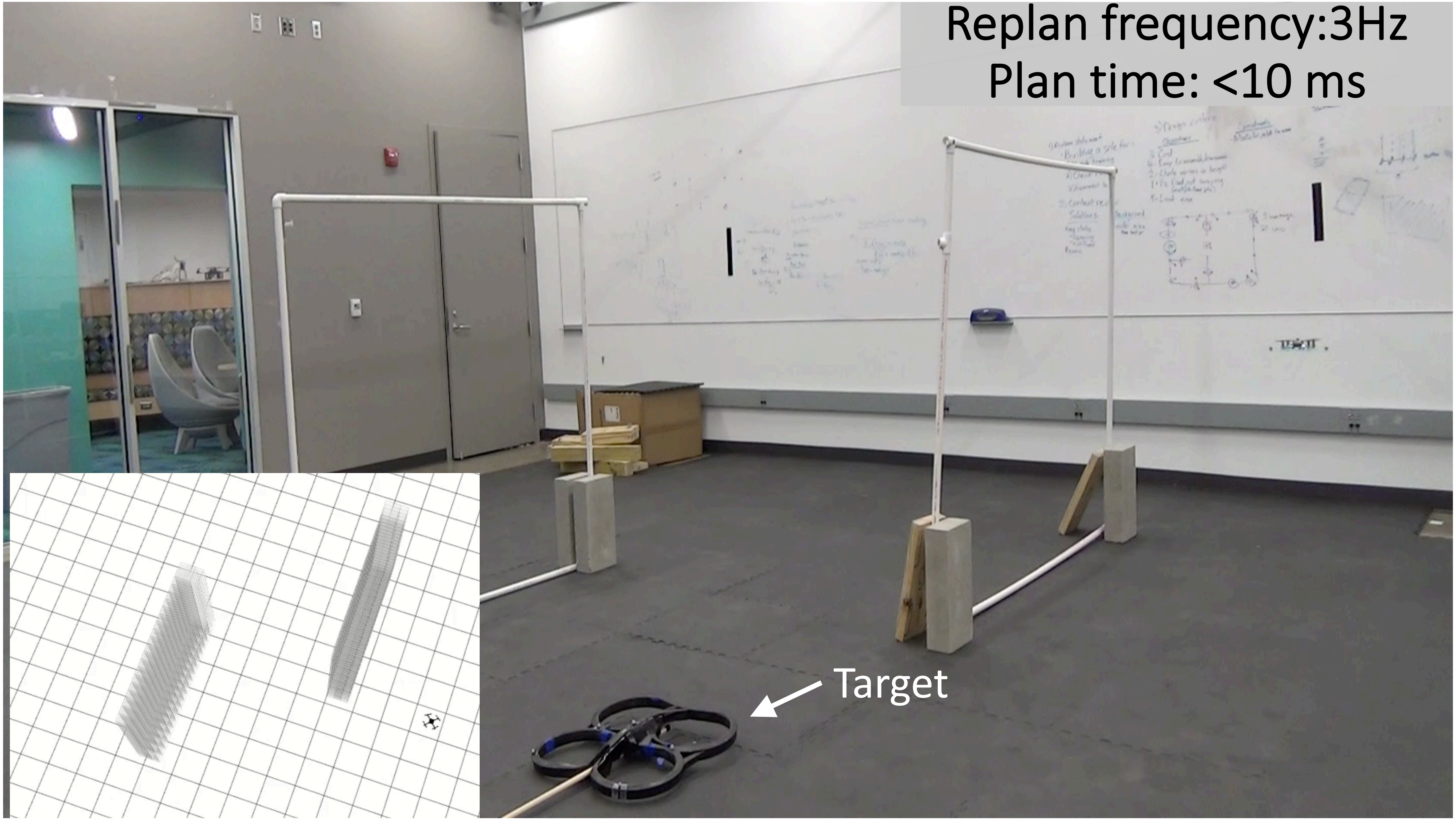
Experiment 3
Tracking a dynamic goal

Target

Moved by human

Quadrotor

Goal is 0.5m above the target


Replan frequency:3Hz
Plan time: <10 ms
Target

Thank you